Cluster state preparation using gates operating at arbitrary success probabilities

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Abstract. Several physical architectures allow for measurement-based quantum computing using sequential preparation of cluster states by means of probabilistic quantum gates. In such an approach, the order in which partial resources are combined to form the final cluster state turns out to be crucially important. We determine the influence of this classical decision process on the expected size of the final cluster. Extending earlier work, we consider different quantum gates operating at various probabilites of success. For finite resources, we employ a computer algebra system to obtain the provably optimal classical control strategy and derive symbolic results for the expected final size of the cluster. We identify two regimes: when the success probability of the elementary gates is high, the influence of the classical control strategy is found to be negligible. In that case, other figures of merit become more relevant. In contrast, for small probabilities of success, the choice of an appropriate strategy is crucial.

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1. Introduction

Measurement-based quantum computing has a number of appealing features not present in the standard gate model. From an experimental perspective, it may well be a significant advantage to abandon the need for exact unitary control between any two constituents, and separate the process of entanglement generation from that of entanglement consumption. In such measurement-based computing, an entangled state is generated, followed by a sequence of local measurements on single constituents. In the original one-way computer [1], this universal resource is the cluster state [1, 2]. In the following, we will indeed concentrate on cluster state preparation; see, however, [3] for a method of constructing a number of novel models for measurement-based computing that make use of resource states different from the cluster state.

Generally speaking, there are two ways of preparing cluster states: on the one hand, this can be done by means of translationally invariant local interactions, not requiring individual local control. This most prominently applies to preparations using cold collisions of atoms in optical lattices by applying spin-dependent shifts [4, 5]. The other way is to build up cluster states from *elementary building blocks*, such as entangled pairs, a framework we will concentrate on in this work. This is the setting that plays the key role when applied to a number of physical architectures: specifically, it is the preferable or typically only applicable method in preparations using linear optical systems [6]–[10], optical systems with weak nonlinearities [11], trapped atoms [12], and matter qubits in optical cavities [13]–[16]. Here, the quantum gates that are applied are typically inherently probabilistic, necessarily leading to a significant overhead in required resources. For linear optical settings in particular, the probabilistic character of quantum gates is unavoidable [17, 18].

There is a new element in this idea that was not present before: *choice*. Indeed, when building up resource states from smaller blocks, several kinds of intermediate structures will appear, and it turns out to be crucial to make a meaningful choice of which parts to attempt to link at what stage. This is no marginal effect, but can give rise to differences in orders of magnitude in, say, consumption of maximally entangled pairs of qubits with state vectors of the form $|0, 0\rangle + |1, 1\rangle$ (referred to as *EPR pairs*). This would apply to maximally entangled photon pairs in the optical context. Such an advantage may be gained even for moderately-sized cluster states [19]. Fortunately, for building up cluster chains using gates with an elementary success probability of the gate of $p_s > 0$, an overhead is sufficient that is linear in the size of the

final chain [10, 19]. For the special case of $p_s = 1/2$ (as, e.g. in linear optical architectures) the consumption of five EPR pairs per edge constitutes an upper bound for the optimal strategy, a bound that cannot be improved any more with sequentially acting gates operating at the success probability dictated by linear optics. Further, for any $p_s > 0$ the overhead required to produce a two-dimensional (2D) cluster out of chains is also only linear in the size of the state to produce. As all these processes are probabilistic, the quoted results actually state that a constant overhead per site is sufficient for any $p_s > 0$ to produce a state of size $n \times n$ (or n in the 1D case) almost certainly as n becomes large [10].

For other elementary probabilities, $p_s > 0$, one may ask: what is the method of choice of dealing with the intrinsic randomness? This is the question that we address in this work. The analysis presented in this work complements recent analytical investigations [10, 19] and numerical work on this topic [20] (see also [21]). In contrast to [10, 19], where also rigorous asymptotic bounds have been presented, we here solely investigate the optimal and worst strategies for finite N using a computer-assisted proof, for arbitrary p_s and different gates. We discuss also the algorithm which is capable of finding the provably optimal classical control strategy and delivers symbolic results for the expected final size of the cluster. Finally, some detailed numbers on the resource consumption in the preparation of 2D cluster states are given.

2. Techniques

In the first part of this work we aim at building up linear cluster chains from a reservoir of N maximally entangled pairs. Our interest lies in identifying the optimal classical control strategy—hence it will be fruitful to abstract from the underlying quantum system. Indeed, at a given point in the process of building up the cluster we have a collection of cluster chains of various lengths at our disposal. The 'state' of our system can hence solely be described by the respective lengths of the chains. This information will be represented by a *configuration* vector $c = (c_1, c_2, \ldots, c_{max})$, where c_i is the number of chains of length i, as counted by the number of *edges*. An EPR pair, for example, has length 1. The question of optimal cluster state preparation then becomes essentially a combinatorical one.

For the task of joining intermediate cluster states to form longer chains, we employ entangling gates like the type-I fusion gate [7], which plays a prominent role in linear optical architectures. We restrict ourselves to two qubit gates that act symmetrically and with the same action for chains of all lengths—most suggested quantum gates do have this property. The effective actions on two chains of lengths l_1 and l_2 in the number of edges can then be described by the outcomes on *success* and *failure*,

$$\{l_1, l_2\} \mapsto \{l_1 + l_2 + d_s\}, \tag{1}$$

$$\{l_1, l_2\} \mapsto \{l_1 - d_f, l_2 - d_f\}, \tag{2}$$

respectively, and are cast into the tuple (d_s, d_f). This family embodies four gates: on the one hand it contains two with an undefined failure outcome. To obtain a proper cluster state, additional Z-measurements on the neighbours have to cutoff the end qubits, thus $d_f = 2$. Additionally, there are those gates with a 'built-in' Z-measurement on failure, $d_f = 1$. On the other hand, there are

Table 1. The four quantum gates described in the text. (d_s, d_f) denote the number of edges gained on success and the number of edges deleted per chain on failure, respectively. p_{NDM} is the probability of success of a photon number non-demolition measurement that has to follow the respective gate.

Gate	d_s	d_f	Physical realization
CZ	1	2	Distant atoms [12, 14]
KLM CZ	1	1	Linear optics with $p_s = 1/16$ [6] or $p_s = 1/4$ [22]; weak non-linearities with $p_s = 3/4$ [11]; linear optics with $p_s = p_{NDM}/9$ [23, 24]; $p_s = 1/8$ [9]
DPC	0	2	Trapped atoms and frequency qubits [13], $p_s < 1/4$
Fusion	0	1	Linear optics parity check [7], optimal $p_s = 1/2$ [10]

two 'parity check' gates (no new edges are created, $d_s = 0$) and controlled-Z (CZ) (entanglement is created, $d_s = 1$). See table 1 for more details.

We can now describe our model for the generation of cluster states. The starting point is a configuration consisting of N EPR pairs. In each step of the process, we choose two chains out of the repository and try to fuse them. The choice of which specific two chains to take is determined by the classical *strategy*. A strategy is hence a complete prescription which chains to fuse for all possible configurations that may occur. This is thought to be a sequential process. Note that if certain parts of a configuration are certainly not used as resources in other steps, one can practically perform some of the steps in parallel. For the theoretical analysis of resource consumption, however, it is always legitimate to think of a sequence of gates that is applied.

The process continues until either (i) there is only a single chain left or (ii) the strategy decides to keep only the longest chain in the current configuration and to disregard the rest. Note that for the case $p_s = 1/2$—of central importance in linear optical architectures—one never benefits from halting before all smaller chains have been consumed [10]. The performance of the strategy can then be measured by the expected length of the longest chain in the final configuration: since this is a probabilistic process, the outcome will in general be an expectation value $\langle L \rangle$ of the final length over all possible final configurations. Note that there are *two different* means involved here: on the one hand, there is the distribution of cluster states of different lengths in a given configuration. On the other hand, since cluster state preparation is a probabilistic process, there is the distribution of configurations in the first place. It is the role of these two different means that renders the discussion of the influence of the classical strategy involved.

We will obviously be interested in the performance of the optimal strategy,

$$Q = \langle L \rangle_{opt}(N), \tag{3}$$

which is the final average chain length under the optimal strategy, which we will also refer to as the *quality* of the configuration consisting of N EPR pairs. We will also pay quite a significant attention to the worst strategy, however, to assess what influence the decision process can possibly have. More concretely, we will ask what are the longest and shortest expected lengths one can obtain out of N initial EPR pairs, if one continues to try to fuse all intermediate resources together until only a single chain is left.
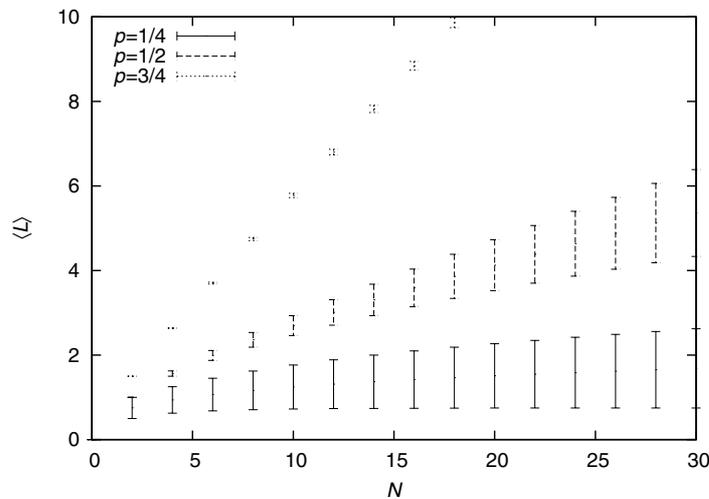
Figure 1. The graphs show the influence of the classical strategy on the expected length of the final cluster for type-I fusion gates operating at $p_s = 1/4, 1/2$ and $3/4$, respectively. For each probability, the range enclosed by the error bars indicates the spread between the best possible and worst possible strategy.

3. Arbitrary gate probabilities

Depending on the physical context, p_s can vary over quite a range of different values. In the *linear optical context*, $p_s = 1/2$ plays a prominent role as the optimal success probability of the parity check gate [10]. When taking inefficient detectors into account, needless to say, this success probability will rapidly decrease below the theoretical value of $p_s = 1/2$.

In [13], probabilistic quantum gates on remote *trapped atom qubits* have been considered, exploiting interference of optical frequency qubits. Here, the success probability p_s is relatively small, smaller than $1/4$ even for perfect photon detectors. However, the respective measurement gate is constructed in a way to be very robust with respect to noise. We will see that for these kinds of gates operating at a relatively low probability of success, the choice of strategy is in fact crucial.

However, gates constructed using *weak nonlinearities* in optical systems as considered in [11] can result in $p_s = 1 - 2^{-1-n}$ where n is the number of ancillas used [11]. In this context, we will see that the worst strategy performs essentially identical to the optimal strategy, hence confirming that in the regime of high success probability, strategic choice hardly matters.

In contrast to other approaches, we give exact values for the optimal and the worst strategy. Furthermore, we will stick to our previous notion of strategies acting on a fixed reservoir of resource states. However, using theorem 16 from [10], these results also deliver the solution to the converse question, of the average resource consumption for construction of a chain of given length (see [20] for numerical results on this converse question, namely the generation of fixed length chains using an infinite supply of entangled pairs).

Figure 1 shows how the optimal and the worst strategy perform when using the type-I fusion gate at different probabilities. Examples from three different *regimes* are shown, the distinctness

of which is quite remarkable.

1. For small gate probabilities the choice of strategy is crucial. Applying gates in a particularly unfortunate fashion might actually result in no net increase of the length of the longest chain at all. However, employing the optimal strategy the length of the longest chain turns out to be an increasing function of the number of EPR pairs, rather than a constant one. For the asymptotic behaviour, this is actually shown in [10].

2. In some intermediate regime (like $p_s = 1/2$ for type-I fusion gate) both the optimal as well as the worst strategy result in a length increase of the longest chain. But still, the choice of strategy distinguishes whether an efficient growth, i.e. a linear growth $O(N)$ will be obtained, in constrast to merely a growth of $O(\sqrt{N})$ (shown in [10]).

3. What may well be considered intuitively plausible is confirmed by the simulations as well: for p_s being large, the difference in the performance of strategies becomes negligible for practical purposes. The order in which the fusion attempts are carried out hence hardly matters. This comes in handy as one does not have to concentrate on the optimal strategy in experimental realizations any more. Even though the optimal strategy also realizes the least storage time in this setting, there is also almost no difference in the storage time required by the different strategies. Then, of course other meaningful figures of merit enter centre stage, like the amount of feed-forward or rerouting needed. Strategies like STATIC [10] that only fuse nearest neighbours in the repository of chains will then be favourable as they realize the least amount of feed-forward in this setting simultaneously. This strategy is already heavily parallelized, in the sense that many fusion operations are applied in parallel rather than waiting after each fusion for the outcome of the previous attempt, decreasing the overall preparation time. Of course, this is another property which is highly desirable and attention can now be centred upon it.

   Minimizing the average storage time as well as the number of applications of the entangling gate are crucial to prevent the system from unnecessary strong decoherence. Depending on the physical implementation a suitable figure of merit in this context could also be the storage time weighted with the chain size.

In this latter setting, there is no need for shuffling chains around, rather than only single qubits. A huge amount of rerouting on the level of qubits is still needed. Kieling *et al* [25] present a possible solution to this problem, making use of tools from percolation theory.

Figures 2 and 3 show the dependence of the standard deviation σ of the final length distributions on the gate probability. As the gate probability increases not only the upper and lower bound to strategy performances converge, but also their relative variances converge and vanish. Interestingly enough, the relative difference between the two strategies gets even smaller than their relative standard deviations.

4. Additional gates

The action of all gates used here can be described by a pair of two parameters (d_s, d_f) (similar to the treatment in [20]): the number of edges d_f that are deleted from the participating chains in case of failure and the number of edges d_s that are added to the resulting chain when the gate is
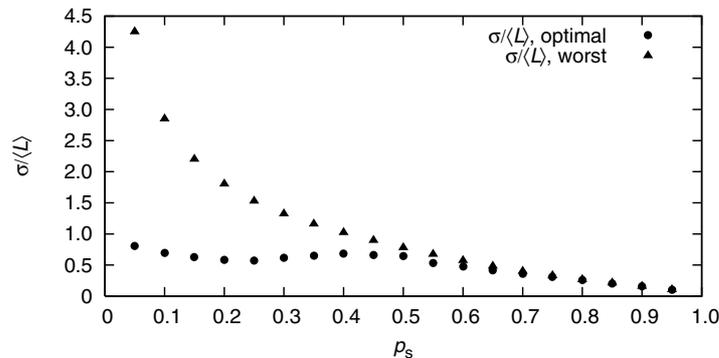
Figure 2. Relative standard deviation $\sigma/\langle L \rangle$ of the distribution of final chains for the optimal and worst strategy respectively. It turns out that for high success probabilities, the statistical fluctuations for a single fixed strategy are more pronounced than the difference between even the best and the worst strategy.
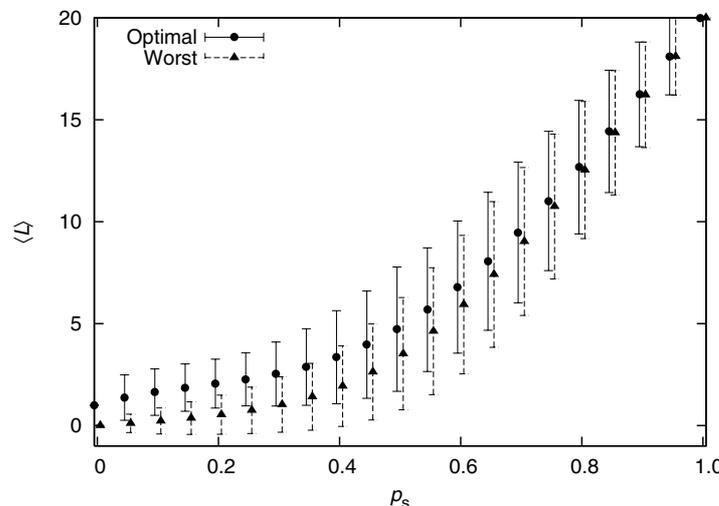
Figure 3. Expected final length of the optimal and the worst strategy, starting from $N = 20$ EPR pairs. Error bars give the standard deviation of the final length distributions. To allow comparison but ensure distinguishability at the same time the data have been shifted by 0.005 to the left and to the right, respectively.

successful. The type-I fusion gate deletes the participating qubits in case of failure. The length of the resulting chain on success is the sum of the lengths of the original chains. Therefore, this gate can be described by the parameters $(1, 0)$. Besides having investigated its performance at different success probabilities, we further compared it to other possible entangling gates. A CZ gate creates an edge between the qubits it is acting on. However, if on failure the outcome is not known [20], the two qubits have to be deleted by applying Z-measurements to the neighbouring sites. Thus, this gate then effectively deletes 2 edges from both chains on failure, and it is represented by $(2, 1)$. The linear optical implementation of the CZ gate (KLM CZ gate [6]) has, in contrast, a defined error outcome which is effectively the action of a Z-measurement on the two qubits. Therefore, this gate simply cuts the two qubits from their chains on failure, and

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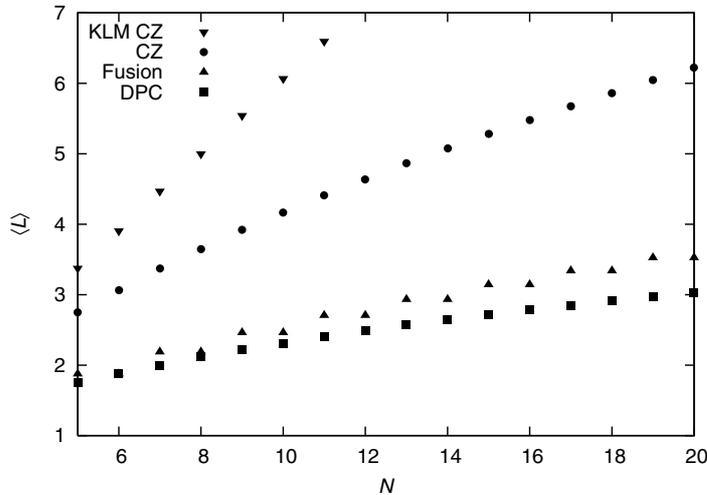
Figure 4. The optimal expected length $\langle L \rangle$ of the final cluster for the gates listed in table 1. For comparison all gates are assessed at $p_s = 1/2$.

consequently is characterized by (1, 1). For the sake of completeness we also introduce a gate which does not add any new edges (like the fusion gate), but deletes two edges from each chain on failure (like the CZ gate), thus resulting in (2, 0). This gate might be a parity check gate, like the fusion gate, but with an undefined failure outcome, thus denoted by DPC (destructive parity check). Actually, the individually trapped atoms can be used to implement this type of gate [13].

Further gates would be for example the type-II fusion [7], or the one created by weak nonlinearities using the qubus technique [11]. However, the first one is hard to compare with the gates presented here as it requires three qubit cluster states (locally equivalent to GHZ states, $|0, 0, 0\rangle + |1, 1, 1\rangle$), rather than EPR pairs to start with. The latter one is actually already included in our analysis, as it corresponds to a gate with parameters (1, 1), operating with a gate-probability $p_s = 3/4$.

The performance $\langle L \rangle_{\text{opt}}$ of the optimal strategy using all four gates is shown in figure 4. The difference between the gates in the number of edges consumed on success and on failure is clearly reflected by the optimal average final length that is achieved. As noted earlier, we do observe an interesting crucial dependence of the performance of the best and worst strategy while varying p_s: 1/2 marks some *intermediate regime* for the type-I fusion gate, above which the choice of strategy does not carry weight anymore. This behaviour is indicated by the diminishing gap in the first part of figure 5. The second part shows this plot using the KLM CZ gate. Interestingly, in this case the worst and optimal strategy are already indistinguishable at gate probabilities far below 1/2. Not only the probability, but also which type of gate is used influences how important the classical choice of strategies will be.

5. Algorithm

Can one use a computer-assisted proof to identify the optimal strategy for a given configuration? Naively, one would expect this not to be the case, as the number of configurations with n entangled bonds is exponential in n [10] and the number of strategies is, in turn, exponential in the number

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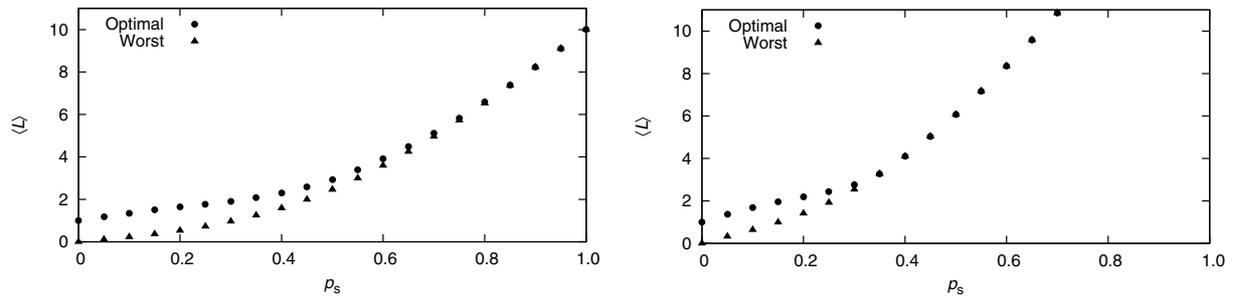
Figure 5. Performance of the optimal and worst strategy for $N = 10$ for type-I fusion (left graph) and the KLM CZ gate (right graph). Both gates have the same number of lost edges on failure.

of configurations. A brute-force search for the optimal strategy is clearly unfeasable. Fortunately, the problem can be addressed using a smarter, recursive algorithm, as a variant of a backtracking algorithm.

To understand how, note that as a result of an attempted fusion, the number of entangled particles decreases by one in the case of success and by at least two in the case of failure. Now consider a given configuration c with n vertices (not edges) and assume that we know the quality Q for all configurations of strictly fewer than n constituents. If the number of chains in c equals l, then there are roughly l^2 possible choices a strategy can make. If the strategy decides on fusing chains i and j, then the expected final length is going to be

$$p_s Q(\text{fuse}(c, i, j)) + (1 - p_s) Q(\text{fail}(c, i, j)). \tag{4}$$

The preceeding value can be computed explicitly, because by assumption Q is known for both fuse(c, i, j) and fail(c, i, j). Hence, using $O(l^2)$ tests, one can identify the optimal pair (i, j).

A computer implementation builds up a lookup table, i.e. a list containing the quality Q of every configuration up to a given number of particles n. The process starts with the trivial configuration $n = 2$, for which the quality is known and works its way up to higher n as described before. Note that the optimal strategy can only be specified by means of a lookup table for all configurations, and there is generally no 'easy description' of the optimal strategy, so a description with a small Kolmogorov complexity.

Figure 6 presents an explicit pseudo-code implementation. By adjusting the parameter p_s and the subroutines fail( ) and fuse( ), the program is easily modified to general gates. Also note that finding the *worst* possible strategy can be achieved similarly.

The usefulness of practical implementations will be limited by memory consumption, as the lookup table grows linearly in the number of configurations, which is exponential in n. We utilized the computer algebra system *Mathematica* to assess the quality of configurations consisting of several dozen particles. A desktop computer completes the calculations in a few hours.

The actual optimal strategy does in fact not have to be unique. However, our investigations indicate that taking the smallest chains most of the time is optimal. Deviations from this rule depend on the success probability and there is no obvious way to describe them with such a small complexity as 'always take the shortest chains'. For some insight into these deviations see [10].

NAME: Optimize
INPUT: Integer n
OUTPUT: For all configurations C with up to n vertices,
              the global variable $Q(C)$ is set to the quality of C.

```
1    SUB Optimize(n)
2      for i := 1 to n
3        C := AllConfs(n)
4        foreach c ∈ C
5          OptimizeConf(c)
6        end for each
7      end for
```

NAME: OptimizeConf
INPUT: Configuration c
ASSUMPTION: For all configurations c' with fewer particles than c,
                    the global variable $Q(c')$ is set to the quality of c'.
OUTPUT: Sets global variable $Q(c)$ to quality of c.

```
1    SUB OptimizeConf(c)
2      l := length(c)
3      for i ≤ l
4        for j < i
5          q[i, j] := p_s Q(fuse( c, i, j )) + (1 − p_s)Q (fail( c, i, j ))
6        end for
7      end for
8      Q(c) := max_{i,j} q[i, j]
```

Figure 6. The recursive algorithm which computes the optimal expected length one can obtain from a given configuration. It relies on three simple sub-routines: AllConfs(n), which returns a table of all possible configurations with up to n particles; fuse(c, i, j), returning the configuration resulting from c after a successful fusion of the ith and jth chain and finally fail(c, i, j) which acts likewise, but assumes the fusion to fail.

However, a worst strategy seems to be, regardless of the success probability, the strategy that always acts on the longest pair of chains.

## 6. Preparation of 2D structures: 'weaving'

We now turn to the preparation of 2D structures, universal for quantum computing. 2D cluster states can be generated starting from linear chains. This can be achieved by placing them on top of each other so that they inherit the square lattice geometry of the cluster states, and by applying entangling gates at the crossings subsequently. This procedure (*weaving*) obviously gives rise to the required 2D structure when all gates succeed. However, sufficiently many spare qubits have to be available for each crossing in case a gate fails.

Here, the probabilistic character of the employed quantum gates is again crucial: one has to make sure that the preparation becomes *almost certain* for large 2D cluster states. Fortunately, it turns out that this aim can always be achieved: as has been shown in [10] (compare also [21] for a strong indication suggesting a polynomial bound), starting from linear cluster chains, 2D cluster states of arbitrary size $n \times n$ can be built almost certainly for large n, with an overhead per site that only depends on the gate probability. Hence, the overall probability of success satisfies

$$P_s(n) \to 1, \tag{5}$$

when using $O(n^2)$ EPR pairs for a state of size $n \times n$. Moreover, this is true for any $p_s > 0$. This is obviously the optimal scaling that can be achieved for 2D cluster states [10].

The gates necessarily act on vertices located along the chain, and not only on vertices at the end of the chain. Hence, the employed quantum gates must provide suitable error outcomes
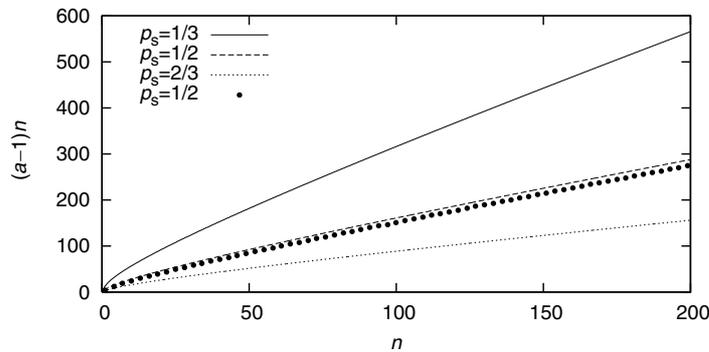
Figure 7. Overhead $(a - 1)n$ in the weaving provess with elementary probability $p_s = 1/3, 1/2, 2/3$ to succeed for cluster size n with a probability of at least $P_s = 19/20$. In addition to the upper bounds, exact values for $p_s = 1/2$ are given.

in order not to tear the chains apart. Then, the weaving procedure as proposed in [19] can be used to construct a cluster using gates succeeding with any $p_s > 0$ almost certainly with a constant overhead per site. This procedure consists of n parallel chains in the horizontal direction and one long chain that connects the short ones in the vertical direction. To find the actual prefactor of the leading quadratic term, we recall the processes that consume edges when constructing a site.

1. The cluster construction itself—there are n^2 sites and $2n(n - 1)$ edges.
2. A σ_z measurement is applied to a qubit on one of the chains, destroying 2 edges.
3. Failures of the fusion gate result in alternating deletions of two edges from the two chains involved.

Fixing an overall success probability results in a number of overhead edges that determines the number of possible failures. Up to a constant error per chain—thus only linear in n—the number of failures equals the number of edges consumed. The number of edges per chain depends linear on the side length of the cluster to be produced, the coefficient being a (see figure 7).

The lower bound for the overall success probability [10]

$$P_s(n) \geqslant \left(1 - \exp\left(-\frac{2(anp_s - n + 1)^2}{an} \right) \right)^n \tag{6}$$

is displayed in figure 8. Chosing $a > 1/p_s$ will result in $P_s(n) \to 1$ as $n \to \infty$. By fixing n and p_s we can extract from (6) a for a given P_s, so also the number of overhead edges that are required to achieve at least this overall success probability,

$$a = 1/p_s + \varepsilon + o(1) \tag{7}$$

with $\varepsilon > 0$. Figure 9 shows the dependence of the required resources on the gate probability p_s. Summing up all these contributions we arrive at a resource consumption when building 2D clusters from existing chains of at least

$$4 + 2(1/p_s - 1) \tag{8}$$

per site, for example 9 edges at $p_s = 1/2$.
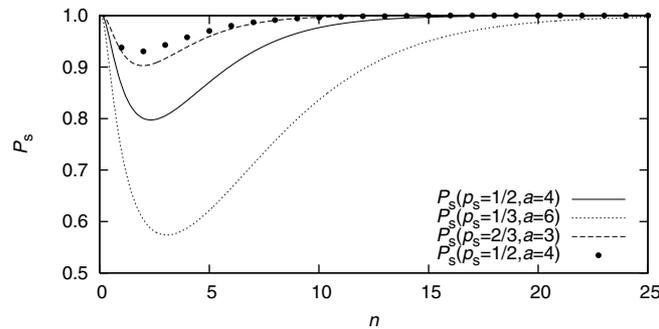
Figure 8. The weaving success probability $P_s(n)$ for fixed gate probability $p_s = 1/3, 1/2, 2/3$ and overhead $a = 2/p_s$. For $p_s = 1/2$ exact values are given for comparison with the lower bound.
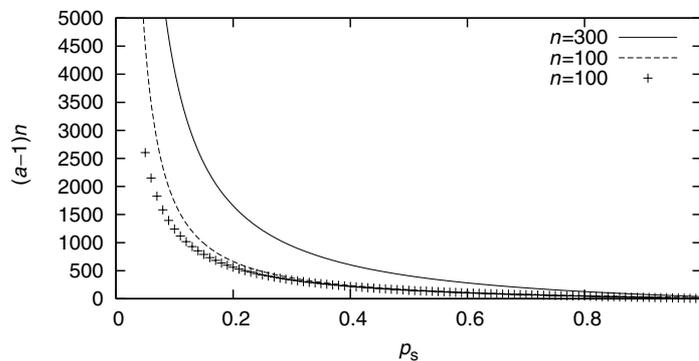
Figure 9. Overhead $(a - 1)n$ that is needed for the weaving process to succeed for a fixed cluster size $n = 100, 300$ with a probability of at least $P_s(n) = 19/20$. For comparison with the upper bounds, exact values are given for $n = 100$ as well.

This prefactor—so far the best known one—can possibly be improved still. For example, one may aim at not preparing a 2D cluster state, but a state that is equivalent to such a state, up to local unitary rotations. Specifically, one could aim at preparing a graph state that is equivalent to a cluster up to local Clifford operations. The orbit under such local Clifford operations is reflected on the level of graphs by the orbit under *local complementations* [26, 27]. Gilbert *et al* [28] already exploit such local complementations when efficiently preparing 2D structures. It would be interesting to see whether systematic explorations of these tools give rise to a significant improvement of the above prefactor in the optimal quadratic scaling.

7. Summary

We have applied the tools introduced in [10, 19] to a number of different quantum gates to prepare cluster states for quantum computing from elementary blocks. Depending on the underlying physical architecture, the gates operate at different success probabilities. A qualitatively different

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behaviour of the difference between the optimal and the worst outcomes has been observed when varying p_s. The specific probability at which this transition occurs depends on the parameters of the gate used, i.e. how many edges are consumed on success and failure. At low probabilities the choice of strategy is highly significant, and any scheme for building up cluster states has to be based on a good choice of a strategy. For gates at high p_s, the potential difference is negligible. Hence, in this regime, other figures of merit, minimizing imperfections and error propagation [29] in an actual experimental context, become the relevant key quantities. This work provides a guideline to what extent the choice of the classical preparation strategy is crucial. Similar ideas could also be applied when building up structures that may be used for fault-tolerant or error resilient schemes [30, 31]. Needless to say, we have concentrated on the preparation of cluster states for measurement-based quantum computing. It would be interesting to see how the new freedom of quantum computing using universal resources different from cluster states as in [3] affects the strategies of preparation.

Acknowledgments

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